Music: A Bridge Between Two Cultures

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Abstract

Certain aspects of European art music occupy a middle ground between the two cultures described by C. P. Snow almost fifty years ago. Analogies exist not only between mathematics and the ratios underlying musical notation and intervals (i.e., the distance between pitches) but also between computer science and counterpoint (simultaneous melodies): in both cases there is a precise syntax of individual parts within the whole.

Music's relationship to the humanities and social sciences is also of considerable interest to bridging Snow's two cultures; trends in music, especially opera, have sometimes seemed to herald events in the social, political, and economic realms.

These and other correspondences suggest that rigorous classroom instruction in music, with an emphasis on its links to other disciplines, should be required throughout the school years and on into the undergraduate core. Such a policy, if implemented, might well help future generations to bridge the divide between Snow's two cultures.

Introduction

C. P. Snow's *The Two Cultures*[1] deals at length with the lack of communication between the world of science and that of the humanities, which he treats almost exclusively in terms of literature. Snow does, however, remark that music is an "important exception" to the general lack of art in the culture of the sciences.[2] Taking this thought a bit further, one might state that the arts in general, and music in particular, have various links to each of the two cultures, and thus offer a potential means of contact between these domains. The following discussion will explore some of the most important connections and suggest ways to make them a vital part of the educational process, not only in the school years but also in undergraduate general education classes. What I am advocating may appear somewhat radical: music should be taught not in isolation but rather in a manner that coordinates with and supports instruction in other subjects. Such a policy would lead to better communication in adult life among individuals pursuing a variety of careers. While the present commentary will focus on classical music, some of the observations may apply to other types of music--folk, jazz, popular genres, even non-western--as well.

Discussion

When I speak of music study, I mean more than just listening or performing, although these activities are vitally important; the direct experience of music transcends language and thus

[1] C. P. Snow 1998.

[2] Ibid. 15.

holds the potential to bring diverse groups together, at least momentarily. (Many school districts do foster cross-cultural understanding with music programs that include singing, and dancing to, music from many different traditions.) This proposal, however, however, will draw on an idea that goes back to late antiquity. At that time music was considered one of the seven liberal arts, within which it was grouped with arithmetic, geometry, and astronomy to form the subdivision known as the quadrivium. In that context music meant what we would now call music theory (the technical and detailed study of how music is put together) with an emphasis on its connections to mathematics--a connection that has remained a source of fascination from the ancient Greeks down to our own time.[3] Plato (427-347 B.C.), writing in the *Republic* (380-370 B.C.), describes how children of the ruling class should be trained in the underlying relationships among music, plane and solid geometry, and astronomy.[4] By contrast, children of the warrior class that would defend the ideal Republic were merely to be exposed to specific types of music that would mold their characters to the desired ends.

The world-view of the ancient Greeks saw music as the embodiment of mathematics, and thus fundamental. In the *Timaeus* (date uncertain) Plato, like Pythagoras (sixth century, B.C.) and his adherents, attributes musical proportions to the universe itself; while in the *Republic* he speaks of a silent "music" produced by the movement of the celestial bodies. This idea of a "music of the spheres" proved to be enormously influential on philosophy and the arts for many centuries;[5] a full treatment of that topic, however, lies beyond the scope of the present paper. Suffice it here merely to state that educated people from late antiquity throughout the Middle Ages and Renaissance, and beyond, were expected to have a basic understanding of "music" in the sense of the quadrivium. (Singing, playing, and dancing were also pursued as cultivated pastimes by the upper classes. However, virtuoso-level performance was left to professional performers, who generally came from the lower classes.)

[3] Rothstein 1995 and Harkleroad 2006 offer wide-ranging introductions to this topic.

[4] Cannon, Johnson, and Waite 1960, 5-25; and Lang, 1941, 13-18. Lang, 17, notes that certain ancient Greek architecture may have incorporated musical-mathematical principles laid out by Pythagoras.

[5] Meyer-Baer 1984; and James1993 provide wide-ranging introduction to the influence of the esoteric Pythagorean tradition from ancient times through the early twentieth century. Ruff and Rodgers 1983 presents an audible version of the music of the spheres as Johannes Kepler (1571-1630) conceived of it.

Today it would seem reasonable to consider "music," in the sense of the seven liberal arts, as having a broader scope than in centuries past. Not only does it relate to mathematics, which is the language of science, but also (whether literally or symbolically) to a variety of other disciplines: computer programming; history, sociology, and philosophy, myth and its symbols, and by way of myth, psychology.

In the tradition of the quadrivium, we can begin by noting a few of the links between music and mathematics. While most of this discussion will focus on instruction at the high-school level and beyond, at least one math-music analogy would be highly useful at a much earlier level: namely, rhythmic notation as an aid to mastering fractions. Rhythmic values are written down according to the same simple ratios that come up in the early study of fractions--e.g., the whole note equals two half-notes, which equal four quarters, etc., while a dot after a note increases its duration by one-half. Learning to recognize note values well enough to clap out simple rhythmic patterns would help children literally to "get the feel" of the arithmetic lesson in terms of sound and movement.

For maximum impact such an exercise should be closely coordinated with the arithmetic lesson--ideally, built into it by the classroom teacher. Similar coordination is equally important at more advanced levels, though it will then usually be best for a specialist to deal with the musical aspects of interdisciplinary topics.

In high school or college, studies in physics or acoustics courses of the properties of the overtone series might well include at least passing reference to wind instruments, particularly the so-called "natural" wind instruments in use till the mid-nineteenth century. Such discussion could then be followed up in the music class. "Natural" horns and trumpets, which continued in use until the middle of the nineteenth century, lacked the valves of their modern counterparts and were limited to producing the pitches available in the "overtone-series." (Overtones are inaudible "partials" above the fundamental pitch that one actually hears.) [6] The type of melody made necessary by the limitations of the overtone series became so associated with these instruments that the idiom of a trumpet fanfare or horn call was easily recognizable even

[6] See *The new Harvard dictionary of Music*, 4th ed. s.v. "Harmonics"; and Piston 1959, 226.

when imitated by other instruments. Many eighteenth-century keyboard sonatas, particularly those by Domenico Scarlatti, (1685-1757), abound with such exchanges of idiom.[7]

To summarize, the relationship between physical law, the construction of "natural" instruments, and the typical sound of music written for them should be made clear to students in coordinated presentations by science and music instructors, rather than scattered piecemeal across curriculum (if present at all) and left to the student to integrate. Such a unified approach would add not only a cultural dimension to the physics or acoustics class but also, and equally important, a scientific one to classroom music instruction. Ideally, these concepts would reach students headed for many different professions across the two cultures.

Counterpoint offers another link between the two cultures for high-school or college students to explore. This technique involves the simultaneous unfolding of two or more melodic lines; the melody in one part of the texture (sum total of parts at any given moment) may imitate the same melody in another part; alternatively, two different melodies may unfold at the same time. A special type of counterpoint known as *fugue* is of particular interest here because it represents a symbolic analogue to computer programming; both undertakings involve independent parts that function in a precise syntax. In fugue a subject (melodic idea) is first presented as a solo in one voice (part), then repeated in other voices of the texture;[8] voices not currently stating the subject present other melodic material known as the countersubject(s). Punctuating complete statements of the subject are so-called episodes, which incorporate fragments of the subject, or counter-subject(s), or both. This description is much simplified; many fugues are quite complicated.[9] However, studying a few reasonably straightforward examples--for example, J. S. Bach's Fugue in C Minor, from *The well-tempered clavier*, Pt. 1 (1722)--in tandem with the early stages of programming courses would add an aesthetic dimension to the learning process. Conversely, a music lecture would acquire a broader (in student parlance, more "relevant") context should the instructor use a discussion of fugue as a springboard to consider symbolic analogies among many kind of systems--not only computer programs but also machines, governmental and business hierarchies, the human body, the

[7] An excellent example occurs in the "horn" call at the beginning of Domenico Scarlatti's Sonata in D Major, K. 96.
[8] In general, fugues consist of two to six voices, with three or four being the most common.
[9] J. S. Bach's sublime collections *The well-tempered clavier* (Pt. 1: 1722; Pt. 2: Leipzig, ca. 1742), and *The art of fugue* (incomplete at Bach's death in 1750) explore fugue's many complexities. See Bach 1989 and Bach 1996.

balances of forces within nature, the universe itself: wherever the function of individual parts is coordinated within a larger unit.

The most rigorous type of counterpoint is canon, in which one voice strictly imitates another for long stretches. If the voice that follows uses exactly the same pitches as the one that leads, the result is canon at the unison. Canonic imitation, however, can play out at any interval above or below the leading voice, producing canon at the second, third, fourth, etc.[10] In addition, the "follower" may be an inversion, retrograde, or retrograde inversion of the "leader": i.e, presenting it upside-down, so to speak, by reversing the direction of each interval in the original version; or backwards (known as "crab" canon; or upside-down and backwards or it may unfold more slowly than the "leader."[11] Particularly astonishing feats of canonic writing occur in double canon, in which two different canons play out at the same time.[12]

Canon is intellectually and musically stimulating, and students at all levels should experience hearing and performing it; appropriate examples run the gamut from "Three Blind Mice," for pre-schoolers, to intricate excepts from Bach's *Musical Offering* in a high-school or college ensemble. However, canon can be exceedingly complex, and detailed study of it is therefore best left to music majors. This is not to say that canon holds no analogies to other disciplines. Highly pleasing ones certainly exist; but appreciating them would require a higher level of sophistication than most of today's undergraduates, much less high-school students, possess. One need only recall Douglas Hofstadter's playful but challenging *Gödel, Escher, and Bach* to appreciate this point.[13]

Hofstadter's comparisons among music, mathematics, and the visual arts are as unexpected as they are apt. A particularly witty discussion early in the book addresses what he calls Strange Loops. These are what occur when we move up or down through "the levels of some hierarchical system [only to] unexpectedly find ourselves right back where we started." By way of illustration, he describes a canon from Bach's *Musical offering*, the *Canon per tonos.*[14] This piece is constructed so as to modulate (shift) nearly imperceptibly to a key a step higher

[10] J. S. Bach's *Goldberg variations* [1741] illustrate canons at the unison through the ninth. See Bach 1981.
[11] Bach illustrates these and other possibilities of canon in his great collection, *The musical offering* (1747). See Bach 1974.
[12] Johannes Ockeghem's (ca. 1420-1497) *Missa prolationum* offers a well-known example of that tour de force. See Ockeghem 1947.
[13] Hofstadter 1980.
[14] Bach 1974.

than the one in which it began. It concludes in the new key, then repeats, beginning in the new key and ending in a key yet a step higher. The process continues until coming full-circle after six repetitions to end in the original key of C minor. Hofstadter points out that a visual equivalent of this canon occurs in the paradoxical (and well-known), "Waterfall" (1961), by M. C. Escher (1902-1972).[15] This lithograph shows water flowing through an edifice that makes several right-angle turns. If viewed in small sections, the water seems to remain on the same plane. However, examination of the structure as a whole shows that this cannot be; the final turn leads to a waterfall that returns the water to its starting-point. As with the modulations in the Bach canon, each move to a higher level in Escher's construct is virtually invisible as it is happens; the unexpected waterfall thus gives us a Strange Loop.[16] To further amuse the reader, Hofstadter notes that the Incompleteness Theorem (1931) formulated by mathematician Kurt Gödel (1906-1978)[17] is also a Strange Loop: In the system of numbering Gödel used to prove this theorem " . . . numbers are made to stand for symbols and sequences of symbols . . . and this coding trick enables statements of number theory to be understood on two different levels: as statements of number theory and also as statements about number theory." [18]

Analogies between music and number also lead us into the esoteric terrain of number symbolism in music. This is a vast subject matter where much has been proposed and little proven;[19] for the most part, therefore, it lies beyond the scope of the present discussion. In some cases, however, certain clearly understandable number symbols could be valuable in college seminars whose subject matter already includes symbolic elements--e.g., art history, literature, or Jungian psychology. For example, the number *three* may appear in such guises as music written for three voices or instruments, a work consisting of three main sections, a basic beat-pattern of three, or sub-division of the beat into three, or a key signature of three flats or sharps, etc. In the music of Christianity *three* has for centuries been associated with the Trinity, and, by extension, the idea of perfection. It may also function as a symbol in secular

[15] Reproduced in Hofstadter, 11.
[16] Ibid, 10-11, full discussion of Strange Loops, 10-23.
[17] Kurt Gödel, "Über formal unentscheidbare Sätze der *Principia mathematica* und verwandter Systeme, I" (On formally undecidable propositions in *Principia mathematica* and related systems), *Monatshefte für Mathematik und Physik* 38 (1931), published in English as *On formally undecidable propositions* (New York: Basic Books, 1962), quoted in Hofstadter, 11, full citation p. 748-49.
[18] Hofstadter, 18; discussion of Gödel's Theorem, 15-19.
[19] *The* new *Grove dictionary of music and musicians*, 2nd. ed., s.v. "Numbers and music," by Ruth Tatlow.

music. A well-known example occurs in *The magic flute* (1791), by Wolfgang Amadeus Mozart (1756-1791), where *three* is prominently associated with Masonry. Masonry, of course, embodied the ideals of the American Founding Fathers and was also a strong influence on some of the leading minds of Europe, including Mozart himself and Johann Wolfgang von Goethe (1749-1832). A brief consideration of *The magic flute* in terms of its "Masonic connection"[20] would add a memorable perspective to college-level discussions of the history of the late eighteenth century, particularly as regards the United States and its philosophical underpinnings.

In a world increasingly permeated at all levels by technology, connections between it and music should be of interest to undergraduates in a variety of majors. Instruction should make clear that over the last two centuries influences between music and technology have gone both ways. In the nineteenth century, as railroads opened up continents, and the industrial revolution fostered the growth of the middle class, the growing focus on technology also resulted in significant improvements in the construction of the piano and of wind instruments. This, in turn, made it possible for composers to demand, and performers produce, dazzling feats of virtuosity. Music in such a vein mirrored the period's underlying sense of the heroic and of the ability of the individual to triumph over adversity. Piano pieces by Franz Liszt (1811-1886) or violin music by Niccolò Paganini (1784-1840)--both of which require a certain "heroism" to play, let alone perform, successfully--illustrate this correspondence with notable panache. How better to communicate the onward-and-upward mindset of that age than by supplementing fact-filled lessons on the political, social, economic trends of the time with appropriate selections of its most brilliant music--which was able to come into existence only because of technical advances in builders' workshops?

The twentieth century saw further collaborations between music and technology, with composers beginning to explore the use of electronically generated sonorities, either alone or in combination with more traditional sounds. It would be helpful to point this out to students in computer-science and music courses alike. Letting them hear a few examples, perhaps even encouraging them to try their hands at writing a short piece themselves, might serve to remind

[20] See Chailley 1992.

them that technology, in addition to the tasks it accomplishes, can also be a means of artistic expression.

Sound-recording technology continues to become increasingly sophisticated, giving us ever-more-realistic approximations of live performance but also placing new stresses on the performers. Artful splicing has made it possible to edit all wrong notes out of commercial recordings, making the products of such wizardry somewhat comparable to the airbrushed photos one sees on the covers of popular magazines. In both cases doctoring creates a standard of perfection virtually impossible to achieve in real life. It would be useful for college students majoring in music, sound-recording technology, merchandising, philosophy, and various other specialties to come together in a seminar to discuss the ethics of all this. Is "perfection" based on sleight-of-hand desirable? If a certain amount of "improvement" is permissible, what are the limits and who will set them?

Links between music and the humanities are especially clear in opera, which combines drama (or comedy) and music. Operatic trends are of particular interest in connection with history because they have sometimes influenced, at times even seemed to foreshadow, political and social events. As high-school and college students consider the political, economic, and social factors leading up to the French Revolution; or the forces driving nationalistic movements in the nineteenth century; or the myriad events in science, the arts, and the political landscape around the turn of the twentieth century, they would profit greatly by also learning something about what people were seeing in theater during the same period: "entertainment" often reflects life itself. In eighteenth-century Europe the plots of many comic opera and intermezzi (short comic skits performed between the acts of operas) spoofed the very group in power at the time: well-to-do older men. More than 50 years before the storming of the Bastille in 1789, G. G. Pergolesi's intermezzo, *La serva padrona*, (The maid as mistress, 1733), told the amusing tale of a sassy servant-girl who successfully contrives to marry her employer and become mistress of the household. The production became a runaway hit, and for the remainder of the century, comic musical theater was populated by women, young people, and servants who regularly outwitted those above them in the social pecking-order.

Many such operas, of course, were adapted from plays that had had difficulties getting past the censors because of their "subversive" content. When the music was of the highest order, the anti-authoritarian impact became even stronger. Audiences remember catchy tunes and

rhythms--and along with them, at least some of the words. W. A. Mozart's *The marriage of Figaro* (1786), is a case in point. The libretto, by Lorenzo da Ponte (1749-1838), is based on a play by Augustin Beaumarchais (1732-1799), *Le mariage de Figaro* (1784). Before its first performance Beaumarchais' work had to overcome the opposition of no less a personage than Louis XVI, who was enraged by its impudence. In a series of hilarious twists and turns, an aristocrat, Count Almaviva, is made to look ridiculous by an alliance consisting of his wife and their servants. Mozart seems to have been attracted to the story for its complicated plot rather than its politics,[21] but the madcap humor and unforgettable tunes resulted in hit with quite a subversive subtext.

In the nineteenth century music and politics came together in various nationalistic movements. A particularly apt example occurs in the third act of Giuseppe Verdi's (1813-1901) opera, *Nabucco* (1842), *Va, pensiero, sull'ali dorate* (Fly, thought, on golden wings). A cry of the Hebrew slaves for freedom, it was understood by Verdi's contrymen as a rallying cry to throw off their Austrian masters. So much did they identify with the "Italianism" they perceived in Verdi's music that they even seized on his name as a convenient acronym for unification under an Italian king: "*Viva VERDI [Victor Emanueel, Rè d'Italia]!"* Purely instrumental works of the type known as "program" music, which describes something beyond the music itself, also played a part in celebrating feelings of patriotism and regional pride. Examples that were widely-known in their own time and remain in the repertory to this day include *The Moldau*, by Bedrich Smetana (1824-1884), which celebrates the beauty of a river in what is now the Czech Republic; *Iberia* Suite, by Issac Albéniz (1860-1909), which incorporates characteristic rhythms and melodic figures from various regions in Spain; and *Peer Gynt* Suite, by Edvard Grieg (1843-1907), which evokes Norwegian folklore.

The operas and music dramas[22] of Richard Wagner (1813-1883) present yet another important cross-disciplinary linkage, this time between music and myth. Operas of the 17th and 18th centuries were often based on myth, but in the nineteenth century this connection reached

[21] Broder 1962, 66-67.

[22] This is the term that Wagner used for his mature works, which he wrote in accordance with his ideal of the *Gesamtkunstwerk* (complete work of art), in which one individual creates text, music, and other aspects of a production of musical theater. In order of their first performances Wagner's music dramas include: *Tristan and Isolde* (1865), *Meistersinger* (1868), the *Ring* (Rheingold [1869], *The valkyries* [1876], *Siegfried* [1876], and *The twilight of the gods* [18761]), and *Parsifal* (1882).

new heights in the operas and music dramas[16] of Richard Wagner (1813-1883). Following the model of the ancient Greek tragedies, Wagner conceived these works on plots of mythic grandeur. With the exception of *Meistersinger* (1868), all of his most important creations--*The Flying Dutchman* (18430, *Tannhäuser* (1861), *Lohengrin*, (1850), the *Ring* cycle (1869-1876), *Tristan and Isolde* (1865), and *Parsifal* (1882)—are based on medieval legends, and their story-lines play out with a mythic sense of tragic inevitability. The rich symbolic content in these works presents a natural springboard to a consideration of other disciplines that involve myth, metaphor, and symbol: psychology, particularly of the Jungian variety, and certain works of art, literature, and poetry. While Wagner's music is far too vast for in-depth coverage in a single course (at any level), appropriate excerpts could do much to enhance undergraduate core courses, even seminars for upperclassmen ina variety of disciplines.

Certain connections between music and other areas of instruction, though perhaps challenging for individuals who lack a background in music theory, are well worth attention in coordinated music and social-studies lectures. For example, from the late 17th century to about the turn of the 20th century, music was organized around a central pitch known as the tonic. Stepwise patterns of whole- and half-steps above the tonic provided the building blocks of both melody and harmony. This sense of secure centeredness was weakened in the late 19th century by the increasing use of chromaticism (pitches outside the major or minor key in force), and decisively challenged in the early years of the 20th century by a number of factors: the use of new scales derived from ethnic music; impressionism, which treats chords and dissonances very differently than in the system based on major and minor; bitonality (simultaneous use of two keys); unpredictable beat patterns; and especially atonality and dodecaphony, both of which avoid any sense of key center.

Coincidentally or otherwise, these changes coincided with profound challenges in other areas to established beliefs, tastes, and the world order itself: Sigmund Freud's recognition of the subconscious mind and its workings, Einstein's Special Theory of Relativity (1905), abstraction in the visual arts, and above all, the beginning of World War I, in 1914. The pounding dissonances and asymmetrical rhythms of Stravinsky's *Rite of spring* (1913) are an excellent metaphor for raw sexuality (unthinkable in polite Victorian society), as well the ugliness and mayhem that the coming War would unleash. So shocked was the audience at the Paris première that it actually rioted. Even now, almost a century later, experiencing the

primitive violence of this work would draw students from all majors into the upheavals of that time in a way that even the best-written textbook cannot.

Music of the twentieth century embodies metaphors for at least two developments in the twentieth century that were diametrically opposed to each other. On the one hand, so-called "serial" music, in which every aspect--pitch, rhythmic values, even dynamics (loudness or softness)--is rigidly laid out in pre-determined successions, is symbolically analogous to the totalitarian regimes of Stalin, Hitler, Pol Pot, and others. But on the other hand, aleatory (chance) music, in which the composer leaves certain aspects of the music to the performer's discretion, also represents an important trend of the 20th century. Aleatory music finds a parallel in the changing codes of etiquette, dress, speech, and behavior since the late 1960s; in all of these areas choices have become much more a matter of individual preference than they once were.

These changes, of course, were brought on by a variety of causes, the most important of which were Vietnam, the civil-rights movement, and the birth-control pill. It goes without saying that popular music, particularly folk and rock, was very much a part of the protest movement and needs to be considered when discussing it. At the same time, however, aleatory music, while much less familiar to most students, is of considerable interest not only because it provides an apt metaphor in sound for the increasing social freedoms that have marked the last forty years (in the West, al least), [23] but also because much of it was actually written well before the trends that, in retrospect, it seems to prefigure.

Conclusion

We have seen that important links exist between music and various other disciplines, and that bringing these commonalities to students' attention could do much to enrich the educational process.[24] Admittedly, students hoping to gain admission to selective colleges tend to cultivate musical performance as a serious sideline, and elite colleges and technical

[23] The foregoing remarks deal only with aleatory music as a twentieth-century phenomenon, leaving out of consideration musical games of the eighteenth century that involved combining various pre-determined segments into a "new" composition. For a more extensive description of this early "aleatory" music, see Harkleroad, 71-81, and Leonard Ratner 1970, 343-63.

[24] Gardner 2004, Ch. 6, "Musical Intelligence," deals with associations between musical ability and abilities in other areas: language, spatial capabilities (in Gardner's view, possibly connected to talent for musical composition), and "bodily or gestural" language (123). In connection with mathematics, Gardner rightly observes that music, despite its links to math, goes beyond it to express emotion (126-27).

schools--e.g., M.I.T. or Georgia Tech--boast excellent orchestras and music electives taught by distinguished scholars. As I have argued, however, classical music--not only in performance but also in the broader sense that I have described--should be treated *everywhere* as a core element in education, acting as a sort of fulcrum in a unified curriculum that continually emphasizes analogies among disciplines. While the better music-appreciation textbooks do point out some parallels between music and the humanities, relatively few of today's students choose a classroom course on classical music as an elective. To make matters worse, policy makers all too often consider music as a pleasant adjunct to "real" education, to be cut back, or even eliminated, when money is scarce. We should be doing exactly the opposite. Linking music to other coursework would offer students a broader, and also more unified, perspective on the many subjects they must master during their school and university years. In later life the results of such training would provide a basis for dialogue with contemporaries in a variety of professions across Snow's two cultures.

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